AFML-TR-73-95

THE CONCEPT OF SCORE OF A RANDOM SAMPLE

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TECHNICAL REPORT AFML-TR-73-95

MAY 1973

AIR FORCE MATERIALS LABORATORY
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract, which was initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals", was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 1 February 1971 to July 1972. The manuscript was submitted by the author for publication in August 1972.

This technical report has been reviewed and is approved.

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ABSTRACT

To any given random sample there may be assigned a number called its score and denoted by $SC(r, N_{os})$, where r = the number of classes into which the space of the random variable has been divided and N_{os} = the number of order statistics actually used. It is easily determined from the sample elements and offers some definite advantages as a test statistic for selecting the most probable population from which the given sample has been drawn. Its decision power tends with increasing r to the largest power attainable for the given sample size. By means of some versatile computer programs the sampling distributions for several combinations of r and N_{os} have been determined. Tables have been prepared from which the probabilities of twelve different hypothetical populations can be immediately read and their acceptability stated.

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INTRODUCTION

When it is required to test whether a set of N sample values $[x_i] = x_1, x_2, \ldots, x_N$ are drawn from a population, defined by a hypothetical distribution function $F(x)$ with known or unknown parameters, we have to choose a test statistic T, which is some function of the sample values. Then we have to calculate T from these values. By means of the sampling distribution of T, which depends upon $F(x)$, it is possible to select a rejection region, corresponding to a preassigned level of significance ε. If the calculated value of T falls within this region, the hypothetical distribution is rejected. The chance of rejecting the true hypothesis is equal to ε.

It is obvious that no function of the sample values can provide more information than the complete set of these values, but, in some cases, much less, so it seems reasonable to postulate that the best test statistic would be the set itself. The N elements x_i may be regarded as the coordinates of a sample point in the euclidean space R_N of N dimensions. The procedure would then be to determine the probability density of the sample points in the space R_N which is characteristic of the hypothetical distribution function under testing, and to select a rejection region of N dimensions.

This rather complicated problem will be studied by use of a new concept called the score of the sample, which will now be defined.

II

THE SCORE OF A SAMPLE

It will be postulated that the sample will not loose its identity, if its elements are permuted, so we may let x_i signify the ordered elements of the sample, that is, its order statistics.

Let us now divide the space of the random variable into r intervals (classes) without common points by prescribing that all these intervals are half-open, closed on the left, except for the largest interval, which is closed.

The class limits may be arbitrarily chosen, but in the present report we will exclusively use the limits

$$x_c = (x_N - x_1) \cdot c/r + x_1 \quad (c = 0.1, \ldots, r) \tag{1}$$

where x_1 is the smallest and x_N the largest of the order statistics x_i.

This choice is identical with the procedure of transforming the sample $[x_i]$ into the sample $[t_i]$ by use of the formula

$$t_i = (x_i - x_1)/(x_N - x_1) \quad (t_1 = 0; t_N = 1) \tag{2}$$

and taking the class limits

$$c/r \qquad (c = 0,1,\ldots,r) \tag{3}$$

To each order statistic t_i will now be assigned a number, called the score of the order statistic, which is equal to the unit figure of the product $r.t_i$. The score of the sample $[x_i]$ denoted by SCX, will be defined as being an N_{os}-figure number with its figures equal to the scores of the order statistics t_i, as numerically illustrated below.

i	x_i	t_i	r = 8		r = 10		r = 100	
			$t.t_i$	score	$r.t_i$	score	$r.t_i$	score
1	1.0	-	-	-	-	-	-	-
2	2.5	0.15	1.20	1	1.5	1	15	15
3	3.8	0.28	2.24	2	2.8	2	28	28
4	4.2	0.32	2.56	2	3.2	3	32	32
5	10.5	0.95	7.60	7	9.5	9	95	95
6	11.0	-	-	-	-	-	-	-

SCX = 1227 1239 15,28,32,95

None of the figures of the sample score SCX will be larger than $(r-1)$. If the values of t_i are given with two decimal places, then $r = 100$ will yield a sample score which provides the total amount of information in the sample. Thus it is possible to reduce the loss of information, when using the score instead of the sample itself, to any desired amount by taking a sufficient large r. For practical reasons r must not be too large, say, equal to or less than ten.

III

THE RELATIONSHIP BETWEEN THE STATISTICS SCX AND VJX

To any given sample $[x_i]$ we may assign also another number, denoted by VJX, which has been described in an earlier report [1]. It is equal to a number with its figures equal to the number v of the N_{os} used order statistics t_i within each of the r classes. It is closely related to the number SCX. In fact, there is a one-to-one correspondence between them, on the condition of equal limits, due to the fact that SCX is an N_{os}-figure number with a non-decreasing sequence of figures and VJX an r-figure number with the sum of its figures equal to N_{os}.

Class No.	$r = 8$, $N_{os} = 4$				
7					
6					
5			x	x	
4					
3		x			
2					
1	x				
0					
	t_2	t_3	t_4	t_5	
SCX = 1355 VJX = 01010200					

Class No.	$r = 4$, $N_{os} = 8$							
3						x	x	x
2				x	x			
1		x	x					
0	x							
	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
SCX = 01122333 VJX = 1223								

These two statistics can be alternatively used but

if $r > N_{os}$ it is more convenient to use SCX

if $r < N_{os}$ it is more convenient to use VJX

IV

FUNDAMENTAL PROPERTIES OF THE STATISTICS $SCR(r, N_{os})$

Let R be a random sample drawn from a specified population also denoted by R. The score of this sample, denoted by $SCR(r, N_{os})$, where r is the number of classes and N_{os} the number of order statistics t, actually used ($N_{os} \leq N - 2$), will now be chosen as the test statistic. It is of the discrete type, since the total mass of its

distribution is concentrated in discrete mass points. The finite number of these points, K_t, is independent of the hypothetical population R and uniquely determined by r and N_{os}, as listed in Table 1. This number is heavily reduced due to the fact that ordered samples are used, because then anyone of the figures of the score must be equal to or larger than the preceding figure. For example, for $r = 10$, $N_{os} = 4$, an unordered sample would have 10,000 points, whereas an ordered sample has only 715 points. Many of these points are empty, that is, they do not carry any probability masses. The set of empty points is different for different hypothetical populations.

The distribution of the statistic $SCR(r,N_{os})$ is completely described by the finite set of probability masses of each mass point, which for a given pair (r,N_{os}) is characteristic of the population R. Evidently the statistic $VJR(r,N_{os})$ has an identical distribution with $SCR(r,N_{os})$ due to the one-to-one correspondence between the two statistics.

The sampling distribution of SCR and VJR can be computed by use of the programs 10/72, 11/72 and 13/72. The latter program is an extension and improvement of the two other. It is capable of computing simultaneously the distributions corresponding to 14 hypothetical populations, samples of sizes $N \leq 50$, and number of mass points $K_t \leq 500$, and, in addition, the 14 test level functions and the 91 combinations of decision powers. This program includes also the possibility of using a part of the order statistics t_i, since it has been observed that the loss in decision power is not large, when using a properly selected part of them. The number of used order statistic is denoted by N_{os}. Recommended sets of eight order statistics for various sample sizes are listed in Table 2. The number eight has been choosen for the reason that it has been found practical to use $N_{os} = 8$; $r = 5$ for sample sizes $N \geq 10$. The number of mass points then becomes 495, independently of the sample size. However, the number of empty points increases rapidly with N. The selection has been made on the conditions that the used set should be symmetrical, the differences between the order numbers as equal as possible, and the smallest differences located at the tails of the sample. A large number of tables have been prepared for stating the decision powers, setting criteria for the rejection of hypothetical populations and selecting the most probable shape parameters, as will be indicated below.

V

DECISION POWER OF SCR AND VJR

In order to demonstrate the effect of r on the decision power, when using all the order statistics t_i of the sample, the following table has been extracted from Table 3.

Decision power DP(0 vs 1) of $SCR(r,N_{os})$ and $VJR(r,N_{os})$

N	N_{os} \ r	2	3	4	5	6	7	8	10
6	4	-	-	43.4	-	44.0	-	45.6	45.7
10	8	55.2	64.0	66.0	67.4	68.9	71.3	-	-
20	18	77.6	87.0	-	-	-	-	-	-
50	48	94.2	99.2	-	-	-	-	-	-

For any given sample size N, the decision power increases asymptotically with r to a value which is believed to yield the largest power attainable, because for a large enough value of r the score of the sample yields all the information provided by the total set of sample elements.

In the series 9,29,39 and 46 the distributions corresponding to the normal dbn $(x=0)$ and the Weibull dbn with $\alpha = 0.01;0.05;0.1(0.1)1.1$ have been computed, and from them the decision power of all combinations of α. The results are listed in Tables 4,5,6,7 for $N=10,20,30,50$, respectively. These tables involve also the estimation powers $EP = dDP/d\alpha$, including in Table 6 also EP for the unsymmetrical set of OS, indicated in Series 40 of Table 3, and in Table 7 the values of EP corresponding to the sample sizes $N=10,20,30$.

The large effect of the sample size on the decision power should be noted. Introducing the concept of decision risk, $DR = 1-DP$ = the chance of making a false decision, it may be said as a rule of thumb that when deciding between the normal $(\alpha=0)$ and exponential $(\alpha=1)$ dbns on the basis of samples of sizes $N=6,10,20,30,50$ the risk will be 1:2,3,10,30,100, respectively.

VI

REJECTION OF AN ASSUMED DISTRIBUTION FUNCTION

When choosing a rejection region it is understood that there will always be a certain chance ε of rejecting the true hypothesis, which is called the level of significance. The chance of accepting a false hypothesis increases with decreasing ε, which, consequently, should not be taken too small.

In the case of SCR its distribution is presented as a table which says how many times v_i out of, say, 10,000, the i:th score has appeared, that is, $v_i/10{,}000$ is the probability that SCR takes its i:th value. A large part of the scores have a probability equal to zero and they should, of course, be included in the rejection region. But we have also to include ε per cent of the remaining scores. Evidently, we will choose those having the smallest probability, that is, the smallest value of v_i. To this purpose, the above mentioned programs have to compute also the test level functions $TL = f(v)$, corresponding to each hypothetical dbn function and defined as follows.

Let n_i be the number of scores having $v = i$,

then TL is defined by

$$TL = \sum_{j=1}^{i} j.n_j = f(i) \qquad (4)$$

Evidently TL is equal to the relative number of scores which have a $v \leq i$ and the scores having a $TL < \varepsilon$ should be included in the rejection region.

VII

SELECTION OF THE MOST PROBABLE DISTRIBUTION FUNCTION

The distributions of VJR corresponding to the Series 9,29,39, and 46 in Table 3 are listed in Tables 8,9,10, and 11 for the Weibull dbn, $\alpha = 0.05;0.1(0.1)1.0$ and the normal dbn (denoted by $\alpha = 0$). The dbns corresponding to $\alpha = 0.01$ and $\alpha = 1.1$ have been excluded because these shape parameters do never appear in connection with fatigue test data. Furthermore, all values of VJR having a $TL < 5\%$ are also excluded.

By use of these tables, the selection becomes extremely simple. The procedure consists in computing VJX of the given

sample X and reading from the table, that value α which yields the largest probability. If the computed value of VJX does not appear in the table, all Weibull and the normal dbns should be rejected, which may be taken as a strong indication that the sample belongs to a two-.or more-component population.

REFERENCE

Weibull,W. "A new test operator, VJ, based on class frequencies." AFML-TR-73-97, May 1973

Table I. Number of mass points K_t of $SCR(r,N_{os})$

N_{os} \ r	2	3	4	5	6	7	8	9	10
2	3	6	10	15	21	28	36	45	55
3	4	10	20	35	56	84	120	165	220
4	5	15	35	70	126	210	330	495	715
5	6	21	56	126	252	462	792	1287	2002
6	7	28	84	210	462	924	1716	3003	5005
7	8	36	120	330	792	1716	3432	6435	11440
8	9	45	165	495	1287	3003	6435	12870	24310
9	10	55	220	715	2002	5005	11440	24310	48620
10	11	66	286	1001	3003	8008			
11	12	78	364	1365	4368	12376			
12	13	91	455	1820	6188				
13	14	105	560	2380	8568				
14	15	120	680	3060	11628				
15	16	136	816	3876					
16	17	153	969	4845					
17	18	171	1140	5985					
18	19	190	1330	7315					
19	20	210	1540	8855					
20	21	231	1771	10626					
21	22	253	2024						
22	23	276	2300						
23	24	300	2600						
24	25	325	2925						
25	26	351	3276						
26	27	378	3654						
27	28	406	4060						
28	29	435	4495						
29	30	465	4960						
30	31	496	5456						
38	39	780	10660						
48	49	1225	20825						

Table II. Recommended sets of eight order statistics selected from sets of size N_{os}

N_{os}	Diff.	Sets of OS
8	1111111	1,2,3,4,5,6,7,8
9	1112111	1,2,3,4,6,7,8,9
10	1121211	1,2,3,5,6,8,9,10
11	1122211	1,2,3,5,7,9,10,11
12	1221221	1,2,4,6,7,9,11,12
13	1222221	1,2,4,6,8,10,12,13
14	2221222	1,3,5,7,8,10,12,14
15	2222222	1,3,5,7,9,11,13,15
16	2223222	1,3,5,7,10,12,14,16
17	2232322	1,3,5,8,10,13,15,17
18	2233322	1,3,5,8,11,14,16,18
19	2332332	1,3,6,9,11,14,17,19
20	2333332	1,3,6,9,12,15,18,20
21	3332333	1,4,7,10,12,15,18,21
22	3333333	1,4,7,10,13,16,19,22
28	4443444	1,5,9,13,16,20,24,28
48	6777776	1,7,14,21,28,35,42,48

Table III. Decision power DP(0 vs. 1) of SCR and VJR

Series No.	N	N_{os}	r	K_t	DP	Program No.	Nr of α	Comp. time	Used order statistics
1	6	4	4	35	43.4	10/72	3	56	All
2	"	"	6	126	44.0	"	"	68	"
3	"	"	8	380	45.6	"	"	96	"
4	"	"	10	715	45.7	"	"	159	"
5	10	8	2	9	55.2	10/72	3	93	All
6	"	"	3	45	64.0	"	"	99	"
7	"	"	4	165	66.0	"	"	117	"
8	"	"	5	495	67.6	"	"	165	"
9	"	"	"	"	67.3	13/72	14	441	"
10	"	"	"	"	67.2	"	3	111	"
11	"	"	6	1287	68.9	10/72	"	279	"
12	"	"	"	"	-	13/72	6	517	"
13	"	"	"	"	-	"	"	423	"
14	"	"	"	"	-	"	3	331	"
15	"	"	"	"	-	"	1	171	"
16	"	"	7	3003	71.3	10/72	3	487	"
17	"	6	6	462	66.2	13/72	3	116	1,2,3,5,7,8
18	"	"	"	"	67.2	"	"	115	-"-
19	"	5	7	462	66.5	"	"	121	1,2,4,6,8
20	"	4	9	495	66.1	"	"	134	1,3,6,8
21	"	4	10	715	66.4	"	"	159	-"-

Table III . (Continued)

Series No.	N	N_{os}	r	K_t	DP	Program No.	Nr of α	Comp. time	Used order statistics
22	20	18	2	19	77.6	11/72	3	210	All
23	"	"	3	190	87.0	"	"	233	"
24	"	"	4	1330	89.6	"	"	381	"
25	"	"	"	"	89.0	13/72	5	304	"
26	"	12	4	455	88.5	"	3	203	1,2,3,5,7,9,10, 12,14,16,17,18
27	"	8	5	495	88.3	"	"	209	1,3,5,8,11,14,16,18
28	"	"	"	"	88.4	"	"	206	-"-
29	"	"	"	"	88.3	"	14	721	-"-
30	"	6	6	462	87.6	"	3	207	1,2,3,4,5,6,
31	"	"	"	"	86.7	"	"	213	7,8,9,10,11,12,
32	"	"	"	"	77.6	"	"	204	13,14,15,16,17,18
33	"	"	"	"	89.0	"	"	206	1,4,8,11,15,18
34	"	"	"	"	89.1	"	"	209	-"-
35	"	5	7	462	88.6	"	"	212	1,5,9,14,18
36	"	4	9	495	88.0	"	"	227	1,7,12,18
37	"	4	10	715	88.6	"	"	249	-"-
38	30	8	5	495	95.9	13/72	3	319	1,5,9,13,16,20,24,28
39	"	"	"	"	96.3	"	"	999	1,5,9,13,16,20,24,28
40	"	"	"	"	96.0	"	14	1013	1,3,5,8,11,14,16,18
41	"	6	6	462	94.7	"	3	322	1,6,11,18,23,28
42	50	48	2	49	94.6	13/72	3	124	All
43	"	"	3	1205	99.0	"	"	165	"
44	"	"	"	"	99.2	"	"	176	"
45	"	"	"	"	99.4	"	"	164	"
46	"	8	5	495	99.1	"	14	376	1,7,14,21,28,35.42,48

Table IV. Decision power of VJR(5,8/10) : all OS used

α \ α	1.10	1.00	.90	.80	.70	.60	.50
1.10	-	10.2	18.7	28.2	38.1	47.6	56.8
1.00	10.2	-	10.7	20.6	30.4	40.8	50.7
.90	18.7	10.7	-	12.3	21.9	33.3	43.5
.80	28.2	20.6	12.3	-	12.6	23.9	34.8
.70	38.1	30.4	21.9	12.6	-	14.3	25.4
.60	47.6	40.8	33.3	23.9	14.3	-	14.8
.50	56.8	50.7	43.5	34.8	25.4	14.8	-
.40	64.9	59.5	53.0	45.4	36.6	26.4	14.8
.30	71.9	67.0	61.5	55.0	47.8	37.8	26.7
.20	77.2	73.2	68.5	63.1	56.6	48.2	38.2
.10	81.4	78.2	74.2	69.6	64.1	57.1	48.2
.05	83.4	80.3	77.0	72.6	67.5	60.9	52.6
.01	84.9	82.2	78.9	74.8	69.9	63.4	55.8
0	72.2	67.3	61.7	55.3	48.1	38.8	28.6
EP =	102.0	104.5	115.0	124.5	134.5	145.5	148.0

α \ α	.40	.30	.20	.10	.05	.01	0
1.10	64.9	71.9	77.2	81.4	83.4	84.9	72.2
1.00	59.5	67.0	73.2	78.2	80.3	82.2	67.3
.90	53.0	61.5	68.5	74.2	77.0	78.9	61.7
.80	45.4	55.0	63.1	69.6	72.6	74.8	55.3
.70	36.6	47.8	56.6	64.1	67.5	69.9	48.1
.60	26.4	37.8	48.2	57.1	60.9	63.4	38.8
.50	14.8	26.7	38.2	48.2	52.6	55.8	28.6
.40	-	16.3	27.8	38.5	43.6	47.2	18.5
.30	16.3	-	16.0	27.6	33.2	37.4	11.1
.20	27.8	16.0	-	16.3	21.6	26.2	15.4
.10	38.5	27.6	16.3	-	9.6	14.6	25.3
.05	43.6	33.2	21.6	9.6	-	8.4	30.6
.01	47.2	37.4	26.2	14.6	8.4	-	34.8
0	18.5	11.1	15.4	25.3	30.6	34.8	-
EP =	155.5	161.5	161.5	172.7	200.0	210.0	-

Table V. Decision power of VJR(5,8/20); i = 1,3,5,8,11,14,16,18

α \ α	1.10	1.00	.90	.80	.70	.60	.50
1.10	-	12.8	25.2	38.6	54.6	68.9	78.2
1.00	12.8	-	13.8	28.9	45.7	60.1	70.6
.90	25.2	13.8	-	16.1	33.0	47.9	61.8
.80	38.6	28.9	16.1	-	17.7	34.7	51.8
.70	54.6	45.7	33.0	17.7	-	19.7	38.4
.60	68.9	60.1	47.9	34.7	19.7	-	21.5
.50	78.2	70.6	61.8	51.8	38.4	21.5	-
.40	85.1	80.7	74.6	66.8	55.6	40.8	22.1
.30	91.0	88.0	83.9	78.1	69.0	57.1	41.2
.20	94.4	92.6	90.0	85.0	78.6	70.1	57.6
.10	96.9	95.6	93.4	90.4	86.0	79.8	70.0
.05	97.6	96.3	94.7	92.6	88.7	83.5	75.3
.01	98.1	97.0	95.7	93.9	90.5	85.9	78.5
0	91.1	88.3	84.7	79.1	70.4	59.6	44.6
EP	128.0	133.0	149.5	169.0	187.0	206.0	218.0

α \ α	.40	.30	.20	.10	.05	.01	0
1.10	85.1	91.0	94.4	96.9	97.6	98.1	91.1
1.00	80.7	88.0	92.6	95.6	96.3	97.0	88.3
.90	74.6	83.9	90.0	93.4	94.7	95.7	84.7
.80	66.8	78.1	85.0	90.4	92.6	93.9	79.1
.70	55.6	69.0	78.6	86.0	88.7	90.5	70.4
.60	40.8	57.1	70.1	79.8	83.5	85.9	59.6
.50	22.1	41.2	57.6	70.0	75.3	78.5	44.6
.40	-	22.4	41.5	57.2	65.0	69.1	27.5
.30	22.4	-	21.9	41.2	49.4	55.3	10.3
.20	41.5	21.9	-	22.1	31.3	38.2	18.2
.10	57.2	41.2	22.1	-	11.6	19.2	36.8
.05	65.0	49.4	31.3	11.6	-	9.7	45.3
.01	69.1	55.3	38.2	19.2	9.7	-	50.9
0	27.5	10.3	18.2	36.8	45.3	50.9	-
EP	222.5	221.5	220.0	224.7	236.7	242.5	-

Table VI. Decision power of VJR(5,8/30); i = 1,5,9,13,16,20,24,28

α \ α	1.10	1.00	.90	.80	.70	.60	.50
1.10	-	15.4	32.4	49.7	66.4	79.3	87.0
1.00	15.4	-	18.0	37.0	55.9	69.8	80.9
.90	32.4	18.0	-	20.4	40.1	58.0	74.6
.80	49.7	37.0	20.4	-	21.9	44.6	64.2
.70	66.4	55.9	40.1	21.9	-	25.4	47.6
.60	79.3	69.8	58.0	44.6	25.4	-	25.6
.50	87.0	80.9	74.6	64.2	47.6	25.6	-
.40	93.6	91.0	86.7	78.3	66.9	50.0	27.9
.30	97.2	95.6	92.0	87.5	80.1	70.0	52.0
.20	98.7	97.8	96.0	92.7	89.3	81.6	69.2
.10	99.3	98.8	97.9	96.6	94.1	89.4	81.3
.05	99.5	99.1	98.5	97.3	75.5	91.8	85.8
.01	99.6	99.4	99.0	98.0	96.5	93.6	89.2
0	97.7	96.3	93.3	88.8	83.0	73.2	57.2
EP	154.0	166.8	191.6	211.1	236.5	255.0	267.5
EP	150.7	161.2	182.1	210.9	231.4	244.5	264.6

α \ α	.40	.30	.20	.10	.05	.01	0
1.10	93.6	97.2	98.7	99.3	99.5	99.6	97.7
1.00	91.0	95.6	97.8	98.8	99.1	99.4	96.3
.90	86.7	92.0	96.0	97.9	98.5	99.0	93.3
.80	78.3	87.5	99.7	96.6	97.3	98.0	88.8
.70	66.9	80.1	89.3	94.1	95.5	96.5	83.0
.60	50.0	70.0	81.6	89.4	91.8	93.6	73.2
.50	27.9	52.0	69.2	81.3	85.8	89.2	57.2
.40	-	27.5	51.6	70.5	76.4	80.5	34.8
.30	27.5	-	28.9	51.3	61.1	67.5	11.5
.20	51.6	28.9	-	27.2	39.4	48.3	22.1
.10	70.5	51.3	27.2	-	14.1	23.8	45.4
.05	76.4	61.1	39.4	14.1	-	11.2	55.2
.01	80.5	67.5	48.3	23.8	11.2	-	62.0
0	34.8	11.5	22.1	45.4	55.2	62.0	-
EP	277.3	282.1	280.8	275.7	281.6	281.0	(1,5,9,13,16,20,24,28)
EP	281.0	285.4	281.2	279.4	281.0	281.5	(1,3,5,8,11,14,16,18)

Table VII. Decision power of VJR(5,8/50);i = 1,7,14,21,28,35,42,48

α \ α	1.10	1.00	.90	.80	.70	.60	.50
1.10	-	19.8	39.0	57.2	74.5	87.8	95.4
1.00	19.8	-	20.8	41.4	65.6	82.9	91.2
.90	39.0	20.8	-	25.6	52.0	73.0	84.7
.80	57.2	41.4	25.6	-	28.8	51.2	75.4
.70	74.5	65.6	52.0	28.8	-	30.6	61.6
.60	87.8	82.9	73.0	51.2	30.6	-	34.8
.50	95.4	91.2	84.7	75.4	61.6	34.8	-
.40	97.2	96.0	94.2	90.8	79.9	60.8	35.6
.30	99.3	98.9	98.0	95.4	90.0	78.8	66.2
.20	99.8	99.6	99.2	97.6	95.4	91.6	83.2
.10	99.9	99.8	99.6	99.0	98.2	96.2	91.0
.05	100.0	99.9	99.7	99.4	98.0	97.2	93.9
.01	100.0	99.9	99.8	99.6	99.2	98.0	95.9
0	99.4	99.1	98.4	95.8	90.6	83.2	70.6
EP =	197.5	202.5	231.5	272.0	297.5	327.0	351.8
EP =	154.0	166.8	191.6	211.1	236.5	255.0	267.5
EP =	128.0	133.0	149.5	169.0	187.0	206.0	218.0
EP =	102.0	104.5	115.0	124.5	134.5	145.5	148.0

α \ α	.40	.30	.20	.10	.05	.01	0
1.10	97.2	99.3	99.8	99.9	100.0	100.0	99.4
1.00	96.0	98.9	99.6	99.8	99.9	99.9	99.1
.90	94.2	98.0	99.2	99.6	99.7	99.8	98.4
.80	90.8	95.4	97.6	99.0	99.4	99.6	95.8
.70	79.9	90.0	95.4	98.2	98.0	99.2	90.6
.60	60.8	78.8	91.6	96.2	97.2	98.0	83.2
.50	35.6	66.2	83.2	91.0	93.9	95.9	70.6
.40	-	37.8	64.2	83.5	88.7	91.4	46.6
.30	37.8	-	39.1	65.3	75.2	81.7	17.4
.20	64.2	39.1	-	34.6	48.4	59.6	28.2
.10	83.5	65.3	34.6	-	19.6	32.0	57.2
.05	88.7	75.2	48.4	19.6	-	15.0	67.4
.01	91.4	81.7	59.6	32.0	15.0	-	74.6
0	46.6	17.4	28.2	57.2	67.4	74.6	-
EP =	366.8	384.2	368.5	361.7	385.6	376.2	-
EP =	277.3	282.1	280.8	275.7	281.6	281.0	(5,8/30)
EP =	222.5	221.5	220.0	224.7	236.7	242.5	(5,8/20)
EP =	155.5	161.5	161.5	172.7	200.0	210.0	(5,8/10)

Table IIX. Sampling distributions of VJR(5,8/10); All order statistics t_i used

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
00404	10	9	4	3	2	0	0	0	0	0	0	3
00512	12	17	21	12	10	5	3	0	0	0	0	12
00521	23	29	23	18	8	4	5	2	0	0	0	14
00530	10	17	17	11	10	3	0	0	0	0	0	16
00611	4	5	9	12	3	3	2	1	0	0	0	4
00620	8	5	6	7	4	5	0	0	0	0	0	7
01061	12	12	9	3	3	0	0	0	0	0	0	4
01232	155	159	130	74	44	24	14	3	2	5	0	94
01250	35	40	32	12	15	7	2	0	0	0	0	17
01313	68	58	61	30	17	18	3	3	1	0	0	38
01322	110	105	109	75	53	28	13	5	5	4	4	85
01340	45	46	45	40	13	12	7	5	0	0	0	39
01403	11	8	15	9	3	1	2	2	0	1	0	9
01421	76	97	80	71	40	32	11	11	8	5	4	60
01430	48	51	58	46	25	15	13	6	4	3	1	30
01511	28	26	30	33	19	6	11	3	4	3	0	34
01520	20	29	45	25	14	10	4	1	4	1	0	27
01610	5	8	12	11	6	6	8	2	2	0	0	10
02051	17	20	11	7	2	4	1	1	0	0	0	5
02132	109	97	72	51	31	17	10	7	3	5	6	72
02141	63	66	62	42	29	11	9	3	0	0	0	35
02150	25	23	20	19	8	4	4	2	2	0	0	10
02213	55	54	61	42	25	11	10	3	1	1	1	33
02222	104	110	100	88	55	39	27	20	11	7	5	118
02231	97	112	104	83	66	42	23	22	15	8	4	82
02240	37	46	47	32	17	12	9	7	6	3	1	28
02303	11	16	18	21	12	9	5	2	3	1	0	11
02312	65	63	57	77	51	33	23	6	5	2	3	62
02321	77	87	111	105	72	48	26	16	14	11	4	98
02330	67	69	80	61	54	21	18	16	6	9	5	59
02402	10	15	16	10	15	8	11	3	2	1	1	17
02411	47	49	58	53	61	32	16	13	9	3	0	59
02420	45	53	49	50	31	26	21	5	4	2	3	78
02501	7	10	23	18	9	8	6	5	4	2	1	16
02510	18	16	25	39	30	20	8	7	1	1	1	33
03023	11	11	21	15	11	8	2	1	1	1	1	7
03032	20	18	13	10	13	5	5	3	2	1	1	17
03041	8	15	20	16	10	5	5	2	1	0	0	8
03104	5	4	8	3	7	5	0	0	0	0	0	2
03113	25	27	14	23	19	7	7	2	0	0	0	22

Table IIX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
03122	30	43	47	51	45	31	19	12	5	4	8	43
03131	38	38	42	48	47	21	10	9	5	2	1	36
03140	18	18	25	15	18	4	8	4	2	2	1	15
03203	13	16	11	5	10	9	4	3	2	1	0	10
03212	38	48	59	47	46	30	16	10	10	7	5	44
03221	55	63	60	83	75	58	39	22	9	4	4	80
03230	40	47	68	57	41	35	18	7	7	4	1	50
03302	15	12	10	15	13	18	10	9	4	3	3	19
03311	40	41	52	71	61	54	36	24	11	6	3	49
03320	38	57	65	75	59	43	30	16	3	4	8	63
03401	3	13	14	25	22	15	12	6	6	3	1	15
03410	22	22	43	42	41	34	21	11	6	6	3	36
03500	2	2	6	6	13	4	4	4	4	2	0	13
04013	5	7	6	5	5	5	4	2	1	1	0	9
04022	10	5	14	12	9	10	8	4	0	0	0	6
04031	9	11	6	6	6	6	3	5	5	3	4	13
04112	15	17	21	22	31	24	14	9	8	5	2	25
04121	20	21	24	28	35	24	17	15	10	6	1	34
04130	10	9	16	29	18	13	15	9	6	4	2	17
04202	6	8	9	9	12	12	9	12	3	1	3	7
04211	26	22	29	47	41	35	37	25	11	10	3	47
04220	19	18	31	36	38	39	24	13	11	10	4	33
04301	8	5	9	11	32	12	11	6	7	4	4	15
04310	7	12	29	45	30	29	32	18	14	6	6	47
04400	4	4	6	14	17	10	9	6	4	3	2	14
05021	5	6	8	3	8	6	7	6	1	1	0	5
05030	1	2	3	7	7	3	3	2	1	0	0	4
05111	9	13	12	13	11	11	9	14	12	8	3	14
05120	12	14	10	15	16	18	14	15	12	5	3	9
05201	0	2	4	9	13	13	13	9	5	3	3	7
05210	5	7	17	32	22	31	23	16	14	9	3	17
05300	2	7	7	3	15	19	15	8	6	5	4	7
06101	0	0	3	4	1	6	7	1	2	2	3	2
06110	4	4	4	4	11	11	9	9	4	3	3	3
06200	0	0	2	5	2	5	7	6	2	4	4	4
10052	24	21	3	2	2	0	0	0	0	0	0	4
10151	32	30	10	7	6	4	4	3	3	2	1	13
10160	9	7	7	3	0	0	0	0	0	0	0	3
10403	12	10	5	3	3	2	1	0	0	0	0	5
10412	36	28	22	19	15	10	9	3	3	3	2	22

Table IIX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
10421	36	43	40	25	33	17	8	6	7	3	3	38
10430	31	26	30	23	12	7	10	2	7	0	0	19
10502	6	3	2	4	7	8	3	3	1	1	1	2
10511	10	17	15	15	5	10	6	2	2	2	2	23
10520	15	26	19	16	12	9	3	2	2	2	2	21
10610	2	6	4	7	5	4	2	2	2	0	0	5
11051	22	28	6	7	6	3	2	0	0	0	0	6
11114	34	26	20	22	14	3	3	3	3	1	1	10
11150	26	26	23	14	7	5	4	1	1	1	0	9
11204	18	15	15	12	11	7	3	1	1	0	0	4
11213	64	73	57	62	31	29	19	9	5	4	3	47
11222	132	108	130	93	89	66	47	33	16	6	5	89
11231	92	96	116	103	76	52	37	21	18	11	6	105
11240	35	49	56	41	35	21	12	12	5	2	5	47
11303	18	18	7	8	15	11	8	6	4	3	2	16
11312	54	68	64	72	53	37	31	28	16	10	6	65
11321	103	101	115	109	106	88	38	28	13	20	12	124
11330	45	57	71	66	55	41	26	20	13	4	3	64
11402	10	11	20	15	24	24	14	9	6	4	4	14
11411	48	61	54	66	48	37	35	15	18	12	13	63
11420	29	41	59	50	61	46	31	22	8	12	7	65
11501	7	7	18	18	10	10	8	7	3	3	3	21
11510	24	17	29	39	31	26	25	10	7	5	4	38
11600	3	5	3	10	8	11	9	4	4	3	2	4
12032	30	36	40	28	20	18	7	6	4	3	1	23
12041	16	22	18	15	16	14	16	7	4	3	1	9
12050	6	4	10	5	3	2	2	1	0	0	0	5
12104	9	8	3	6	9	10	7	3	1	1	1	3
12113	33	35	35	34	32	29	16	20	9	3	3	34
12122	82	85	96	101	64	58	45	33	20	14	7	79
12140	26	20	36	45	33	23	24	15	7	7	3	29
12212	60	77	97	89	86	80	61	41	38	22	16	81
12221	111	115	135	143	125	125	92	74	61	30	18	132
12230	61	64	74	115	81	77	43	25	23	15	11	79
12302	20	19	23	30	31	35	34	21	13	13	9	32
12311	41	56	79	101	114	98	102	57	47	39	31	106
12320	53	72	103	115	120	104	77	52	40	19	12	94
12401	16	19	30	53	35	40	29	21	16	10	6	34
12410	40	47	59	65	93	72	52	34	31	22	7	69

Table IIX.(Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
12500	9	14	17	18	36	28	19	11	7	6	6	23
13013	8	10	10	9	12	8	9	6	5	0	0	11
13022	20	26	19	24	23	18	19	12	10	5	3	23
13031	14	15	20	22	22	22	9	12	14	8	9	21
13040	10	8	5	9	8	4	3	2	2	1	0	10
13103	10	10	11	15	14	11	13	12	8	5	4	14
13112	37	44	48	44	50	51	39	34	34	22	15	44
13121	57	73	63	84	82	76	65	63	34	32	25	56
13130	21	27	44	51	44	46	39	21	15	11	10	60
13202	12	13	20	31	33	32	32	29	27	20	16	29
13211	34	48	82	96	119	120	98	86	57	47	36	91
13220	34	48	71	100	94	99	84	71	59	40	30	91
13301	13	18	28	38	53	58	43	46	32	22	11	38
13310	29	50	74	105	133	119	89	74	54	36	26	97
13400	8	8	29	37	32	48	51	39	22	16	12	32
14012	4	8	11	13	15	17	15	9	5	11	12	9
14021	7	11	11	26	30	26	29	15	13	5	5	18
14030	10	12	7	4	15	16	14	16	12	6	2	7
14102	3	5	8	12	17	21	18	23	14	9	9	13
14111	18	20	27	47	56	50	57	56	46	41	29	34
14120	11	18	28	46	70	60	57	55	29	30	18	41
14201	15	17	19	29	35	41	52	50	37	28	29	30
14210	24	23	41	59	90	97	115	81	62	46	31	73
14300	7	14	16	32	55	64	55	52	39	19	18	36
15002	1	1	0	1	0	3	5	9	7	6	7	1
15011	3	3	7	10	14	12	7	12	11	12	8	8
15020	2	5	4	9	14	15	13	18	7	5	8	7
15101	3	4	13	14	18	23	26	22	25	18	9	20
15110	9	11	16	26	41	52	56	45	34	23	15	25
15200	5	5	16	23	39	49	55	44	32	26	21	28
16010	1	1	3	8	10	12	9	16	10	7	6	5
16100	2	6	2	7	17	14	21	17	21	12	12	8
17000	0	0	0	0	2	8	3	5	1	3	4	1
20033	7	8	10	9	4	4	2	2	2	0	0	5
20042	7	7	3	4	5	1	3	0	1	1	1	3
20132	25	31	17	18	14	11	10	9	5	2	2	14
20141	14	13	20	14	10	6	5	3	3	3	3	18

Table IIX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
20213	17	12	24	15	15	10	8	7	7	2	1	10
20231	28	35	35	29	32	26	21	17	13	10	3	30
20240	9	8	13	10	3	9	4	7	5	2	3	12
20303	3	6	10	7	4	2	4	2	2	0	0	2
20312	18	18	24	26	28	24	13	10	6	10	5	18
20321	25	30	33	41	40	28	26	18	19	15	12	25
20330	16	20	20	16	24	15	9	7	3	6	3	14
20402	7	9	4	6	6	6	1	3	5	3	1	6
20411	18	20	20	20	28	20	13	5	11	7	11	13
20420	12	10	13	12	15	11	13	12	10	7	7	20
20501	4	6	2	4	9	5	6	1	0	1	2	4
20510	6	8	10	7	6	9	14	9	6	3	1	12
21014	10	10	5	2	2	0	2	3	1	2	1	1
21032	25	21	8	12	12	11	6	2	1	3	0	15
21041	11	9	8	12	7	10	2	7	5	4	0	10
21104	1	4	8	8	5	2	3	2	2	1	0	5
21122	45	42	54	51	60	49	39	33	27	13	11	47
21131	44	48	55	59	49	59	38	32	35	31	23	44
21140	13	22	22	23	21	19	17	14	10	5	7	13
21203	7	7	11	11	11	10	9	10	12	10	7	8
21212	54	52	50	62	48	62	68	50	29	28	20	70
21221	49	67	85	85	94	98	84	70	56	67	56	93
21230	31	35	51	53	58	58	55	34	30	29	15	43
21302	16	10	17	19	20	20	30	25	21	18	15	27
21311	44	43	56	74	83	78	59	44	43	22	32	80
21320	31	31	45	68	83	75	63	45	44	31	20	58
21401	10	12	19	32	28	33	35	26	19	14	9	12
21410	24	24	39	36	44	59	52	47	33	26	13	41
21500	5	5	6	16	17	17	21	23	16	14	8	16
22013	8	6	9	10	14	11	11	15	11	11	4	7
22022	17	19	14	15	16	16	24	24	22	23	14	24
22031	18	22	18	18	29	23	28	20	19	23	13	17
22040	9	7	6	7	12	12	6	8	5	6	9	9
22103	7	7	13	19	21	21	18	18	12	7	6	7
22112	30	41	33	48	58	64	76	70	58	41	35	43
22121	48	46	72	90	94	92	95	93	88	69	57	80
22130	23	31	35	54	57	60	52	52	38	33	26	52

Table IIX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
22202	15	16	17	22	37	40	42	51	35	29	27	20
22211	53	72	75	97	125	142	149	124	107	93	71	112
22220	34	45	69	92	123	140	125	121	99	75	66	82
22301	13	15	30	37	44	55	70	79	56	48	26	29
22310	33	48	52	77	95	134	131	109	87	65	68	91
22400	15	21	28	37	41	48	55	57	47	38	24	33
23012	8	5	10	17	17	23	21	24	19	23	21	14
23021	15	15	15	15	28	42	44	38	41	26	20	18
23030	10	12	15	12	13	14	16	27	24	19	13	17
23111	38	41	40	53	89	113	117	115	113	94	83	57
23120	22	22	38	73	82	110	105	99	103	87	68	57
23201	13	20	32	39	71	70	72	82	85	92	66	37
23210	37	38	58	81	121	154	158	175	150	119	90	100
23300	7	12	32	51	70	83	111	85	98	62	55	42
24002	1	1	2	6	5	4	5	10	15	10	12	8
24011	4	6	12	9	21	29	41	34	29	23	33	14
24020	4	6	8	14	21	23	37	32	29	31	28	15
24101	2	5	7	18	28	51	59	66	59	59	52	22
24110	13	23	34	49	71	88	111	99	133	95	78	43
24200	5	3	22	36	62	87	102	112	93	78	58	52
25001	2	2	3	3	8	7	16	24	23	22	17	4
25010	3	3	11	15	17	33	39	39	39	33	17	10
25100	2	2	7	24	38	52	64	76	75	80	55	14
26000	0	0	1	2	2	11	22	27	25	21	24	6
30122	17	22	13	14	10	14	12	9	8	4	5	9
30131	12	8	7	13	7	9	9	11	8	16	14	10
30212	5	7	18	18	14	10	19	22	14	12	10	11
30221	13	18	16	16	24	27	29	21	18	17	19	16
30230	6	6	15	12	15	12	17	13	12	8	10	10
30302	2	0	3	6	6	5	7	7	2	9	6	6
30311	7	9	12	13	15	35	24	28	17	15	12	22
30320	9	8	9	16	25	31	24	19	16	17	8	13
30401	2	1	4	4	8	7	7	9	10	10	6	6
30410	5	7	8	11	11	10	17	22	16	12	12	8
31013	3	4	5	7	3	2	3	5	6	9	12	6
31022	11	13	12	8	11	9	14	14	17	15	14	6
31031	3	4	11	14	16	15	17	16	11	8	9	5
31103	3	2	2	5	5	9	9	6	7	9	6	4
31112	14	15	21	22	31	36	26	37	41	34	32	21

Table IIX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
31121	24	23	27	39	50	52	56	55	60	62	46	36
31130	6	9	17	17	22	28	36	30	34	32	28	21
31202	8	6	7	7	14	19	25	30	39	29	30	19
31211	20	22	28	49	68	84	96	103	100	97	90	42
31220	16	19	34	48	71	67	100	81	76	76	79	35
31301	8	13	11	17	25	42	61	63	63	46	40	16
31310	10	15	26	39	55	72	60	76	80	78	70	40
31400	3	2	6	16	23	26	33	45	34	29	22	17
32012	4	5	3	9	17	25	22	27	23	25	19	9
32021	5	9	14	19	19	25	37	35	40	36	43	16
32030	3	5	7	14	18	22	24	24	20	21	17	11
32102	4	9	7	8	19	26	33	44	40	55	41	10
32111	25	26	42	46	47	68	115	134	141	136	132	39
32120	19	27	34	45	61	87	104	121	128	113	107	48
32201	6	9	22	24	38	60	73	97	108	122	127	24
32210	18	19	42	62	88	135	159	202	204	176	166	55
32300	8	13	14	32	44	82	82	134	120	131	108	33
33011	5	5	12	17	28	30	42	49	64	62	66	17
33020	2	3	6	9	18	36	43	52	47	58	54	15
33101	2	4	6	12	33	51	76	85	106	110	100	21
33110	12	14	24	48	62	100	141	175	197	205	204	48
33200	8	12	16	30	70	101	158	185	195	201	177	35
34001	2	3	1	6	5	19	19	34	51	56	44	3
34010	1	3	8	14	26	39	61	77	80	84	81	15
34100	4	7	12	22	49	86	130	150	163	163	181	23
35000	3	3	4	6	6	20	29	51	66	60	50	8
40130	1	1	2	3	3	7	13	13	10	8	5	3
40202	2	3	3	5	4	6	8	10	11	11	11	0
40220	3	4	3	7	12	12	24	25	26	23	26	7
40400	2	4	2	6	9	10	11	11	12	9	10	3
41021	3	4	6	5	8	17	19	24	23	37	34	3
41030	2	2	1	3	6	8	9	11	18	20	24	3
41201	0	0	2	9	14	27	35	52	74	88	91	12
42020	1	1	4	4	8	19	28	50	69	57	61	7
42200	3	3	5	18	44	63	112	149	203	222	234	32
50102	0	0	1	0	0	0	6	8	11	11	8	0

Table IX. Sampling distributions of VJR(5,8/20) i = 1,3,5,8,11,14,16,18

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
00341	18	24	4	1	0	0	0	0	0	0	0	1
00413	8	9	10	1	0	0	0	0	0	0	0	4
00431	6	9	4	2	0	0	0	0	0	0	0	5
01241	70	87	50	10	1	0	0	0	0	0	0	32
01313	112	100	68	28	6	1	0	0	0	0	0	44
01322	146	171	131	57	10	0	0	0	0	0	0	81
01331	86	98	92	34	11	0	0	0	0	0	0	61
01412	14	18	17	4	3	1	0	0	0	0	0	16
01421	26	22	34	19	1	3	0	0	0	0	0	39
01430	3	5	11	1	1	0	0	0	0	0	0	5
02132	133	134	86	42	10	3	1	0	0	0	0	56
02141	29	35	17	16	4	1	0	0	0	0	0	15
02213	114	126	92	47	25	6	0	0	0	0	0	71
02222	202	188	185	104	34	5	1	0	0	0	0	131
02231	109	132	143	78	33	1	0	0	0	0	0	114
02240	8	12	9	6	4	1	0	0	0	0	0	8
02312	28	44	44	54	16	5	2	0	0	0	0	55
02321	57	65	88	90	24	8	1	0	0	0	0	99
02330	14	14	15	17	6	2	1	0	0	0	0	22
03113	39	31	34	22	16	6	2	0	0	0	0	19
03122	75	75	75	63	28	13	5	2	1	0	0	68
03131	31	38	48	47	18	7	1	0	0	0	0	46
03140	5	8	12	7	3	0	0	0	0	0	0	2
03212	24	35	52	56	45	20	6	0	0	0	0	64
03221	40	68	122	104	88	21	6	1	0	0	0	147
03230	10	11	29	22	13	4	1	0	0	0	0	33
03311	6	9	21	37	28	6	1	1	0	0	0	39
03320	2	6	6	14	10	3	1	0	0	0	0	18
04112	1	1	3	9	10	8	5	1	0	0	0	11
04121	5	7	17	22	17	14	4	1	0	0	0	19
04211	1	4	4	18	15	9	2	1	1	0	0	29
04220	0	1	7	10	6	6	0	0	0	0	0	8
10313	46	60	52	20	10	4	2	0	0	0	0	22
10322	105	99	74	36	14	3	0	0	0	0	0	55
10331	73	74	50	15	9	1	0	0	0	0	0	33
10412	14	15	15	14	2	1	0	0	0	0	0	10
10421	15	18	25	13	2	1	0	0	0	0	0	15
11222	435	440	371	255	116	26	10	2	2	1	0	274
11231	275	284	245	152	62	22	8	1	1	0	0	199

Table IX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
11240	32	34	17	17	6	1	1	0	0	0	0	12
11303	9	13	15	11	7	7	1	1	0	0	0	5
11312	84	101	101	105	71	26	9	1	0	0	0	121
11321	161	198	211	158	89	33	7	2	1	0	0	196
11330	25	28	56	36	17	5	2	0	0	0	0	37
11411	10	11	23	22	4	4	1	0	0	0	0	19
11420	3	6	13	7	12	3	0	0	0	0	0	16
12014	12	6	5	3	1	1	1	1	0	0	0	2
12113	169	181	175	130	84	59	21	12	5	1	0	97
12122	367	357	419	356	224	112	51	19	7	1	0	310
12131	234	285	268	244	156	78	26	9	6	0	0	227
12140	22	30	32	21	10	7	2	0	0	0	0	26
12203	21	24	27	35	25	13	11	4	2	2	0	39
12212	199	250	351	350	311	189	74	41	12	4	7	325
12221	319	414	666	710	475	286	126	44	15	4	3	687
12230	70	100	126	156	101	47	19	6	0	0	0	138
12302	5	9	12	15	17	18	5	2	2	0	0	17
12311	42	81	131	199	176	93	44	21	4	2	1	193
12320	17	29	74	109	101	48	14	5	1	0	0	89
12401	2	5	5	16	20	9	2	1	0	0	0	24
12410	6	7	24	28	27	20	3	2	1	0	0	36
13022	11	16	18	30	20	18	9	6	2	0	0	15
13031	11	13	16	12	18	10	4	2	1	1	0	11
13112	36	49	75	125	136	97	65	43	26	7	5	95
13121	49	90	171	254	292	231	135	54	16	10	7	236
13130	9	22	54	60	74	32	22	10	1	0	0	43
13202	4	8	12	16	24	34	20	8	5	0	0	9
13211	22	37	102	206	249	198	109	60	22	9	2	183
13220	19	20	55	101	102	101	54	21	5	2	0	87
13301	1	4	14	29	48	56	15	6	6	2	0	31
13310	6	10	23	58	83	51	31	9	1	1	1	59
14021	1	1	1	7	9	12	11	2	2	1	1	8
14111	1	3	5	21	47	52	48	29	6	3	1	32
14120	0	5	5	19	25	23	15	7	6	1	0	13
14201	0	0	2	11	25	26	28	5	4	0	0	11
14210	1	1	11	22	43	46	23	11	9	6	1	19
20222	44	43	40	29	17	5	1	1	1	0	0	24
20312	12	13	14	14	13	7	2	1	0	0	0	10
20321	15	11	20	20	11	10	4	2	0	0	0	18
21023	16	18	22	11	4	1	1	0	0	0	0	5
21032	15	12	11	5	5	1	0	0	0	0	0	9
21122	160	164	147	145	128	91	54	24	13	6	3	139
21131	118	100	129	112	88	56	22	11	3	3	1	81

Table IX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
21140	8	12	11	11	8	2	2	2	0	0	0	6
21203	11	19	20	19	18	13	12	8	3	1	0	27
21212	112	145	195	225	205	159	112	67	23	14	5	170
21221	188	237	310	376	424	312	190	100	52	19	11	330
21230	29	44	76	81	70	39	26	11	3	2	0	75
21302	5	5	9	15	19	14	12	7	0	0	0	10
21311	35	51	87	103	135	105	76	38	26	8	2	111
21320	22	23	45	59	51	44	24	19	8	1	0	54
21401	0	1	8	8	10	24	11	4	0	0	0	11
21410	0	5	12	17	17	15	9	4	4	2	1	20
22013	5	7	9	10	14	10	6	4	3	3	2	5
22022	14	18	28	30	28	26	22	15	11	7	1	18
22031	6	18	19	24	30	31	19	9	4	0	0	19
22103	8	7	7	17	18	19	12	8	5	4	1	8
22112	55	80	111	156	213	244	192	133	90	53	29	127
22121	83	119	236	394	456	500	359	244	145	74	36	332
22130	25	25	51	71	97	79	63	29	20	4	3	78
22202	4	11	24	35	44	55	59	38	12	9	3	28
22211	36	65	153	289	441	478	405	292	156	81	34	256
22220	23	40	88	175	229	241	173	104	60	27	8	149
22301	3	5	26	47	99	96	86	45	31	9	6	49
22310	9	17	39	122	157	151	107	60	35	16	5	101
22400	2	2	7	9	11	9	12	6	1	0	0	6
23012	2	2	5	6	15	18	19	24	14	13	9	9
23021	4	7	16	19	42	62	61	43	35	20	14	17
23030	0	1	3	8	10	10	8	10	6	5	1	3
23102	0	0	4	6	12	25	27	29	14	8	8	4
23111	4	7	26	76	143	207	216	159	140	71	30	70
23120	2	7	16	48	95	103	122	73	34	22	14	41
23201	1	1	9	32	70	130	134	91	50	34	14	33
23210	7	10	28	71	166	224	194	140	71	36	18	64
23300	0	1	4	12	25	36	24	27	12	4	2	16
24101	0	0	0	3	5	17	20	16	13	9	6	4
24110	0	0	2	7	20	36	42	21	17	6	1	5
24200	0	0	1	1	6	15	20	8	7	4	0	4
30122	8	13	14	15	17	16	15	10	5	2	1	5
30131	10	11	12	8	8	8	6	4	2	2	1	6
30212	7	8	14	26	20	20	16	20	15	6	7	24
30221	10	23	41	32	39	40	29	30	22	16	7	35
30311	2	3	9	7	20	27	23	16	6	4	2	13
30320	0	2	3	7	7	12	9	2	1	0	0	7
31022	1	4	6	9	15	11	15	8	7	6	4	9
31031	1	2	6	10	11	6	12	8	5	2	1	2

Table IX. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
31112	22	22	44	67	108	138	171	146	131	122	101	54
31121	49	56	92	161	233	296	359	298	223	185	120	119
31130	6	5	8	23	43	52	47	42	25	16	12	25
31202	3	5	10	16	30	48	49	61	49	29	19	13
31211	18	25	58	133	240	339	415	387	312	237	161	113
31220	13	15	33	61	96	152	157	131	105	76	46	55
31301	3	2	11	28	60	81	108	90	65	60	28	27
31310	3	5	23	36	84	112	144	118	74	50	33	44
32012	1	2	3	7	16	37	46	51	51	41	32	9
32021	2	8	12	25	56	90	125	150	131	91	66	20
32030	2	2	4	2	11	15	27	32	16	15	14	8
32102	0	2	6	11	22	43	65	103	99	78	54	6
32111	7	15	32	85	202	419	616	710	705	595	459	83
32120	7	9	18	41	106	228	283	319	277	202	135	44
32201	1	2	10	42	113	234	366	473	420	317	236	46
32210	2	8	27	86	206	400	562	568	487	338	226	85
32300	0	0	4	13	41	71	105	106	97	67	28	15
33011	0	0	0	1	5	21	31	61	58	63	54	2
33020	0	0	1	2	4	22	24	38	29	23	24	2
33101	0	0	2	3	20	56	123	164	178	157	119	6
33110	0	0	7	16	50	136	240	295	258	199	113	13
33200	0	0	1	10	29	63	83	118	100	86	63	13
34001	0	0	0	0	1	2	7	15	15	18	20	0
34010	0	0	0	3	4	13	21	30	39	26	25	2
34100	0	0	0	3	9	29	53	42	51	45	32	0
40121	0	0	1	1	1	6	14	22	24	25	23	0
40211	0	0	1	5	5	14	23	34	44	43	34	5
41012	0	0	1	1	1	3	8	20	24	30	32	0
41021	0	0	2	4	6	14	25	41	61	67	76	4
41111	0	0	4	8	32	62	155	284	403	526	539	9
41120	1	1	1	7	9	38	83	112	154	168	175	6
41201	0	0	0	8	17	44	121	211	315	361	395	5
41210	0	0	6	10	24	68	182	286	398	418	406	12
41300	0	1	1	3	14	28	37	53	77	80	75	1
42020	0	0	0	0	1	6	17	48	62	77	73	1
42101	0	0	0	0	10	33	99	196	350	471	482	2
42110	0	0	0	3	20	67	173	393	546	646	666	4
42200	0	0	0	2	10	47	117	181	240	246	243	4
43100	0	0	0	1	9	25	90	172	224	278	254	0
44000	0	0	0	0	1	5	9	27	24	30	27	0

Table X. Sampling distributions of VJ(5,8/30)
i = 1,5,9,13,16,20,24,28

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
00332	16	28	10	0	0	0	0	0	0	0	0	4
01241	145	147	98	8	0	0	0	0	0	0	0	48
01322	95	112	107	26	3	0	0	0	0	0	0	74
01331	104	113	111	35	1	1	0	0	0	0	0	86
01412	7	13	21	8	3	0	0	0	0	0	0	21
01421	35	43	42	30	6	0	0	0	0	0	0	58
02132	116	110	81	23	3	0	0	0	0	0	0	36
02213	12	10	10	1	0	0	0	0	0	0	0	5
02222	81	100	100	61	17	1	0	0	0	0	0	91
02231	84	114	114	67	7	0	0	0	0	0	0	84
02312	12	30	42	43	8	2	0	0	0	0	0	41
02321	48	91	146	112	33	5	0	0	0	0	0	179
02411	6	7	22	36	17	1	0	0	0	0	0	55
03221	11	19	40	30	26	4	1	0	0	0	0	54
03311	3	6	19	27	17	7	1	0	0	0	0	40
10241	200	201	77	11	1	0	0	0	0	0	0	38
10313	22	18	14	4	0	0	0	0	0	0	0	3
10322	194	220	151	65	11	1	0	0	0	0	0	68
10331	148	162	129	52	8	1	0	0	0	0	0	86
10412	18	19	20	19	7	1	0	0	0	0	0	19
10421	49	84	68	38	6	1	0	0	0	0	0	58
11141	204	205	110	44	5	1	0	0	0	0	0	52
11222	650	741	690	362	134	33	6	1	1	0	0	388
11231	557	615	647	342	101	17	2	0	0	0	0	392
11312	220	268	381	313	148	51	8	0	0	0	0	268
11321	528	741	1022	837	363	79	13	4	0	0	0	912
11330	19	39	44	31	9	2	0	0	0	0	0	36
11411	66	104	238	211	123	44	10	1	0	0	0	274
11420	7	17	40	30	10	4	2	0	0	0	0	51
12122	94	146	149	133	90	36	8	0	0	0	0	106
12131	100	100	158	126	51	16	7	1	0	0	0	93
12212	131	165	238	350	304	171	74	17	2	1	0	264
12221	365	507	905	1164	900	416	119	34	5	1	0	1058
12230	30	31	54	50	27	8	1	0	0	0	0	60
12302	9	12	19	35	48	35	11	5	1	0	0	21
12311	124	227	524	958	948	441	158	40	8	5	0	906
12320	20	35	104	183	119	44	11	2	0	0	0	178
12401	3	6	22	54	51	39	13	5	1	0	0	67
12410	3	5	31	42	43	17	2	0	0	0	0	62

Table X. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
13112	6	6	17	19	32	40	18	7	2	0	0	17
13121	20	33	69	113	143	125	61	13	9	1	1	100
13202	0	1	3	10	23	20	12	4	1	0	0	11
13211	35	55	171	384	603	498	226	81	23	8	1	363
13220	8	13	53	89	77	59	17	4	3	0	0	79
13301	3	4	31	73	123	83	42	16	3	1	1	67
13310	0	12	20	78	92	50	18	3	1	0	0	64
14111	1	4	10	52	115	132	75	27	9	6	1	60
14120	0	2	7	16	38	23	11	6	2	0	0	16
14201	0	0	11	26	64	85	66	21	11	2	2	33
14210	0	0	13	45	78	65	29	8	0	0	0	42
20312	13	15	13	10	7	1	1	0	0	0	0	6
20321	30	35	37	35	17	4	2	1	0	0	0	37
21122	20	22	25	36	33	22	9	2	2	0	0	19
21131	20	29	36	22	22	12	8	0	0	0	0	8
21212	32	49	70	98	104	79	59	31	11	4	2	67
21221	103	151	241	333	342	279	121	72	34	5	1	232
21302	5	6	12	10	19	25	13	7	4	0	0	8
21311	48	81	198	345	382	355	196	90	42	11	2	283
21320	7	13	34	50	57	39	13	4	2	0	0	31
21401	3	4	8	24	31	33	9	4	3	1	0	24
21410	2	4	7	27	23	12	5	4	0	0	0	12
22112	7	7	15	27	55	71	61	40	19	12	5	24
22121	25	34	57	133	238	252	197	111	56	25	11	94
22202	3	4	5	18	22	43	60	28	14	7	4	11
22211	37	66	217	460	840	1056	967	536	251	104	38	336
22220	10	14	37	102	165	135	85	33	18	13	1	88
22301	3	8	36	72	184	245	188	106	40	20	5	72
22310	2	5	28	68	152	134	73	32	16	2	1	66
23021	0	1	3	4	12	40	46	33	13	5	1	3
23102	0	0	1	3	11	13	22	32	15	10	2	2
23111	8	14	44	180	422	764	890	695	385	175	75	132
23120	1	4	20	43	94	147	130	82	28	11	4	34
23201	3	5	24	93	332	539	640	448	234	98	37	90
23210	1	1	28	122	328	460	386	184	85	31	8	119
23300	0	0	0	3	15	23	5	5	5	0	0	7
24011	0	0	0	0	5	20	41	39	32	20	4	2
24101	0	0	0	7	43	138	200	186	95	45	23	13

Table X. (Continued)

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
24110	1	1	2	28	82	202	203	128	58	31	12	18
24200	0	0	0	2	10	21	23	12	5	2	0	2
31121	0	1	7	8	14	30	27	26	26	26	13	1
31211	1	1	8	41	74	128	178	179	160	92	55	16
31220	1	1	2	3	9	18	21	8	4	6	7	2
31301	0	1	3	9	16	31	57	41	34	17	10	6
32111	0	3	11	37	140	333	612	829	833	667	466	30
32120	1	1	3	8	20	59	88	105	70	45	31	7
32201	0	1	4	30	92	286	478	762	685	457	298	19
32210	1	2	6	33	99	254	385	393	279	193	107	16
33011	0	0	2	2	10	36	101	153	185	174	134	3
33020	0	0	0	0	2	8	16	27	29	24	15	1
33101	0	0	0	10	42	191	488	756	866	715	426	5
33110	1	1	3	12	84	270	575	760	642	385	235	13
33200	0	0	0	3	10	55	85	78	69	46	21	2
34001	0	0	0	0	0	10	32	50	48	43	29	0
34010	0	0	0	0	8	21	63	82	91	55	39	0
34100	0	0	0	1	6	16	56	57	31	28	19	1
41111	0	0	0	0	4	14	30	93	149	207	234	0
41201	0	0	0	1	3	21	50	88	150	199	200	1
41210	0	0	0	1	2	10	32	51	97	90	66	2
42101	0	0	0	1	5	46	149	428	787	1006	1059	0
42110	0	0	1	2	12	72	210	465	649	742	629	0
42200	0	0	0	0	3	9	42	78	113	87	73	0
43001	0	0	0	0	2	5	29	88	141	201	201	0
43010	0	0	0	0	1	24	71	171	233	255	235	1
43100	0	0	0	0	3	26	54	137	162	145	87	0

Table XI. Sampling distribution of VJR(5,8/50)
i = 1,7,14,21,28,35,42,48

VJ \ α	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	0
10322	45	67	30	11	0	0	0	0	0	0	0	18
11222	264	312	295	99	17	1	0	0	0	0	0	139
11231	158	211	185	56	12	0	0	0	0	0	0	84
11312	33	44	72	62	12	4	0	0	0	0	0	62
11321	73	127	234	128	27	1	0	0	0	0	0	203
12122	22	34	49	46	20	5	1	0	0	0	0	23
12131	18	26	43	33	10	2	0	0	0	0	0	25
12212	11	23	60	105	70	31	1	0	0	0	0	64
12221	44	93	268	343	191	47	5	0	0	0	0	302
12311	6	18	82	208	137	27	5	1	0	0	0	209
13121	2	4	10	28	47	15	5	1	0	0	0	17
13211	1	1	19	93	165	73	21	3	0	0	0	89
21221	17	28	72	112	90	38	15	2	1	1	0	68
21311	6	7	36	77	71	37	14	4	0	0	0	49
22121	3	9	22	43	80	77	43	19	2	0	0	24
22211	7	12	38	185	408	496	256	84	30	3	1	126
22301	0	0	2	15	49	55	18	7	2	0	0	19
23111	0	1	4	14	95	206	204	114	25	7	1	14
23201	0	0	2	9	61	136	105	32	9	3	0	11
23210	0	0	0	10	58	74	53	12	5	1	0	8
31211	0	0	2	6	9	49	57	49	33	14	3	3
32111	0	0	1	5	26	98	251	213	243	127	62	2
32201	0	0	0	3	24	94	195	229	139	62	33	3
32210	0	0	0	5	11	60	87	67	41	17	6	3
33101	0	0	0	1	7	38	140	192	145	66	27	2
33110	0	0	0	1	10	49	130	111	57	17	6	2
41111	0	0	0	0	0	0	5	20	61	76	69	0
41201	0	0	0	0	0	8	21	37	58	64	42	0
42101	0	0	0	0	0	6	56	187	354	448	383	0
42110	0	0	0	0	1	11	49	191	229	199	148	0
43010	0	0	0	0	0	0	21	24	45	48	22	0
52100	0	0	0	0	0	0	2	14	44	67	79	0

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Professor Waloddi Weibull Ch. Fontanettaz 15 1012 Lausanne, Switzerland	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

THE CONCEPT OF SCORE OF A RANDOM SAMPLE

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(First name, middle initial, last name)*

Waloddi Weibull

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1973	30	1

8a. CONTRACT OR GRANT NO. F 44620-72-C-0028 b. PROJECT NO. 7351 c. Task No. 735106 d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* AFML-TR-73-95

10. DISTRIBUTION STATEMENT

Approved for Public Release; Distribution Unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Air Force Materials Laboratory (LL) Wright-Patterson AFB, Ohio 45433

13. ABSTRACT

To any given random sample there may be assigned a number called its score and denoted by $SC(r,N_{os})$, where r = the number of classes into which the space of the random variable has been divided and N_{os} = the number of order statistics actually used. It is easily determined from the sample elements and offers some definite advantages as a test statistic for selecting the most probable population from which the given sample has been drawn. Its decision power tends with increasing r to the largest power attainable for the given sample size. By means of some versatile computer programs the sampling distributions for several combinations of r and N_{os} have been determined. Tables have been prepared from which the probabilities of twelve different hypothetical populations can be immediately read and their acceptability stated.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Statistics						
Random Sampling						
Sample Population						
Probability						

☆U.S.Government Printing Office: 1973 — 759-495/613

Security Classification